PLANETARY, LUNAR AND SOLAR POSITIONS
601 B.C. TO A.D. 1
AT FIVE-DAY AND TEN-DAY INTERVALS

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INTRODUCTION

These tables are an ephemeris giving geocentric positions (tropic celestial longitudes and latitudes) of the Sun, Moon, and naked-eye planets, to an accuracy and spacing suitable for hisotrical purposes, for the period 601 B.C. to A.D. 1.

The ephemeris is based on the theories of Leverrrier for the Sun and inner planets, Gaillot for Jupiter and Saturn, and Hansen for the Moon, with some elements as modified by Schoch. Th units of tabular precision are 0º.01 for the Sun and planets, and 0º.1 for the Moon. Apart from the final rounding of 0º.005[1] for Sun and planets, and 0º.05 for the Moon, the above theories are rpesented (becasue of the neglect of some smaller perturbations) in longiotude to within 0º.003 for the Sun, 0º.010 for Mercury, 0º.155 for Venus and Mars, 0º.025 for Jupiter, 0º.155 for Saturn, and 0º.021 for the Moon; and in latitude to wqell under 0º.005 for the Sun and planets, and within 0º.077 for the Moon.

Positions are given at 5-day intervals for Mercury, Venus, and the Moon, and ten-day intervals for the Sun, Mars, Jupiter, and Saturn, for 7 p.m. local mean time 45º E. longtitude (Babylon). With these intervals, positions for intermediate times, and other geographic longitudes, can be found by interpolation to virtually full tabular precision, except occasionally slightly less precison for Mercury, and a few 0º.1 for the Moon.

This project was proposed by Professor Otto Neugebauer of Brown University and the Institute of Advanced Study. It was commenced at the Electronic Computer Project of the Institute of Advanced Study, and completed at the Research Center of the International Business Machines Corporation, using about forty hours on an IBM 704 computer.

For the use of the tables, only section 1 is needed. The remainder of the text contains discussion of the methods used, accuracy, differential corrections, etc.

SOURCES

The proposer of this project suggested using as a basis the "Tafeln der Sonne, Planeten und Mond" (vol. 2 of *Tafeln zur astronomisschen Chronologie*) by P.V. Neugebauer, 1914 (to be referred to as PVN (T)), as augmented by improved elements quoted in his *Astronomischem Chronologie*, 1929 (PVN (A)). These books, a standard reference in historical astronomy, contain a number of tables, such as contributions to various elements over different time intervals, equations of center, etc., by means of which a single planetary position for a particular instant can be hand-calculated in perhaps a half-hour; and have proved invaluable for reference and checking purposes.

It was found desirable, however, to refer to the original sources – Leverrier for the Sun, Mercury, Venus and Mars; Gaillot for Jupiter and Saturn; and Hansen for the Moon; with modified elements by Schoch – which PVN used.

The major elements and frameworks of these sources were used, with computaitonal procedures adapted to an electronic computer. Among the numerous perturbations, a selection was made, using the same ones as did PVN for Jupiter, Saturn and the Moon, and more for other bodies. Details are given in sections 2 and 3.

The *Connaissance de Temps*, 1953 and 1954, was also very useful for reference and assistance in checking.

ACKNOWLEDGEMENTS

For their continued support and encouragement I thank the administration of the Institute for Advanced Study[2] and the International Business Machines Corporation (IBM), in particular Dr. H.H. Goldstein of both insitutions; Dr. E.R. Piore of IBM; Dr. J. Robert Oppenheimer and Dr. Hans J. Maehly of the Institute for Advanced Study; and Dr.s L.P. Horowitz, J.M. Berger, and Willard Bouricius of IBM.

My thanks go to Professor Otto Neugebauer for suggesting and advising on this project; to Professor A. Sachs for further advice; to Mr. B. Tasini for programming the perturbation and output routines; to him and to Miss Jean Keller for making test runs on the 704; to Mrs. Mildred Goldberger for desk calculations used in checking; and the computer personnle of the above institutions.

For the loan of astronomical books I thank the Princeton University Observatory.

My wife I thank for constant encouragement.

[1] Small digits, as the 5 here, represent figure beyond the tabular precision for the body being discussed, such as guard figures, rounding errors, etc.

[2] The part of the work at the Institute for Advanced Study was jointly supported by the Army, the Office of Naval Research and the Air Research and Development Command, under Contract No. DA-36-034-ORD-1646, Project No. TB3-0538.

1. USE OF THIS EPHEMERIS

1.1. ARRANGEMENT OF THE EPHEMERIS

Each page contains the (tropic)[1]geocentric longitudes and latitudes, in degrees and decimal fractions, of the Sun, Moon, and naked-eye planets (except the latitude of the Sun, which is zero) for two consecutive years. The commencement of each year, starting with -600 and progressing to + 1, is indicated by its number in three columns across the page, at the top of the page for even-numbered years, and at the middle for odd-numbered years.

To the left of the middle of each page is a column of months, with a column of dates on each side. The dates on the left, at ten-day intervals, are called *major dates*; [2] those on the right are five days later, and are called *minor dates.* For all, 7 p.m. local civil time 45º East longitude, or 4 p.m. Universal Time, is implied.

The positions of Saturn, Jupiter, Mars, and the Sun are given for each major date, on the left of the page (the same side as the corresponding date).

The positions of the Moon, Venus, and Mercury are given for both major and minor dates, that is, at five-day intervals, on the right of the page. The two longitudes for each body are adjacent, and in the same left-right relationship as the dates, that is, the entry for the major date on the left, and for the minor date on the right; and similarly for the latitudes. Thus the longitudes of the Moon for Jan. 3, Jan. 8, .Jan. 13 of -600 are 331º.3, 33º.1, 105º.0 respectively. This arrangement was chosen for compactness. For a close view of the behavior of one of these quantities, as for smoothness or for higher-order interpolation, it is advisable to copy the successive entries into a single column, thus:

ø
331.3
33.1
105.0

The sequence of the planets on the page, from the outermost, Saturn, on the left, to the innermost, Mercury, on the right, facilitates these associations of major and minor dates. The central location of the Sun and Moon facilitates comparisons of their longitudes, whose difference indicates the .phase of the Moon.

A line-space is inserted every two months for legibiIitv.

If major and minor dates on the same line belong to the same month, or if only one is present, owing to spacing, the month is shown; but if major and minor dates on the same line belong to different months, this is denoted by / /. This leads the eye in correctly associating the major date on the left with the month on the line above, and the minor date on the right with the month on the line below.

[1] I.e., with respect to the mean equinox of date.

[2] They are, specifically, the dates for which the Julian day numbers of the preceding Greenwich noons are multiples of 10: that of the first entry, 3 Jan. -600, is 1,501,910.

1.2. THE CALENDAR

The calendar used throughout is the Julian (every fourth year a leap year), which is standard for historical work.

The negative year numbers are in the mathematical and astronomical convention, differing by 1 from the common convention for B.C. years in which there is no year 0. Thus

- 600 = 601B.C.
....
- 100 = 101 B.C.
- 99 = 100 B.C.
....
-2 = 3 B.C.
-1 = 2 B.C.
0 = 1 B.C.
+I = A.D.1

etc. The advantage of the negative year convention is that the number of years elapsed between two dates can be correctly computed by subtracting one from the other algebraically, even if they differ in sign. Thus, from January 1, -2 to January 1, + 2 is (2) - (-2) = 4 years. If the equivalent dates January 1, 3 B.C. and January 1, A.D. 2 were used, the number of elapsed years would have to be computed by subtracting 1 from the sum 3 + 2 of the B.C. and A.D. years.

1.3. TIME RELATIONSHIPS

In describing the use of the tables, several related time scales will be used. Each is the local civil time of the event at some geographic location, and any two differ only by an additive constant.

The *local civil time* (L.C.T.), *t* local, at a particular locality is expressed as a date (of any convenient calendar, say the Julian), plus the elapsed time, from 0h to 24h, since the local mean midnight which commenced that date.

Greenich civil time (G.C.T.), *t* Grw, or Universal time (U.T.), is the particular local civil time of Greenwich (longitude 0º).

If geographic longitudes are regarded as positive when East and negative when West, then the following relationships hold among *t* Grw, *t* local, at a particular place, and the geographic longitude of that place:

$$t\text{ local} = t\text{ Grw} + (\text{geographic longitude}^{\circ}/\ 15^{\circ})\text{ h},$$
$$t\text{ Grw} = t\text{ local} - (\text{geographic longitude}^{\circ}/\ 15^{\circ})\text{ h}.$$

For the nominal longitude of Babylon we adopt 45º East longitude (which is very near the actual value), and call the corresponding local civil time *Babylon civil time* (B.C.T.), *t* Bab. Then

$$t\text{ Bab} = t\text{ Grw} + 3\text{ h}$$
$$t\text{ Grw} = t\text{ Bab} - 3\text{h}$$

The tabular instants, i.e., the instants for which positions are given in these tables, are all for 19h (7 p.m. near sunset), Babylon civil time, i.e., for *t* Bab = Dd + 19h. This instant was chosen at Professor Otto Neugebauer's suggestion as a convenient reference time for the Babylonfan material at hand. It is equivalent to *t* Grw = Dd + 16h (4 p.m., Greenwich).

For interpolation, it is convenient to express the desired instants, on a scale, t tab, in which the tabular instants are integral days *t* tab = Dd, the indicated dates with no hour parts. To do this it is merely necessary to define

$$t\text{ tab} = t\text{ Bab} - 19\text{h}$$
$$= t\text{ Grw} - 16\text{h}.$$[3]

Example 1. *Babylon*. Let the desired instant be 1 A.D., B.C.T., 15 Jan., -600, i.e.,

$$t\text{ Bab} = 15\text{ Jan., } -600, +1\text{h}.$$

Then

$$t\text{ tab} = t\text{ Bab} - 19\text{h}$$
$$= 15\text{ Jan., } -600, -18\text{h}$$
$$= 14\text{ Jan., } -600, +6\text{h},$$

i.e., the desired instant is la6h later than the majortabular date of 13 Jan., -600 (which is the nearest preceding tabular instant for both ten-day and five-day bodies).

Example 2. *A different geographic location*. Let the desired instant be PVN's test date:[4] October 27, 7 B.C., 6 p.m., Memphis. Then

$$t\text{ local} = t\text{ Memphis} = 27\text{ Oct.,} -6(\text{y}), +18\text{h}.$$

Taking 31º 21', East = +31º.35 for the longitude of Memphis,

$$t\text{ Grw} = t\text{ Memphis} - (+31^{\circ}.35/15^{\circ})\text{ h}$$
$$= t\text{ Memphis} - 2\text{h}.09$$
$$= 27\text{ Oct., } -6, +15\text{h}.91.$$

Hence

t tab = *t* Grw - 16h
= 27 Oct., - 6, - 0h.09
= 26 Oct., - 6, + 23h.91,

which is 5d 23h.91 later than the nearest preceding major tabular date of 21 Oct., .-6 (for ten-day bodies), and 0d 23h.91 later than the nearest preceding minor tabular date of 26 Oct., -6 (for five-day bodies).

1.4. INTERPOLATION

If a position of some body is desired at an instan intermediate between tabular instants, it is necessary to interpolate.

Two conflicting design goals for the tables were (1 interpolability with negligible loss of accuracy - say an interpolation error of 1/2 *e*, where *e* is the unit of precision of the tables; and (2) a reasonable size for the table covering the period of interest.

Goal (2) required that tabular instants be spaced nc closer than about 5d. With such a spacing, ordinary linear interpolation has errors far in excess of the desired limit (although it will suffice in many cases where only approximate values are needed). Higher-order interpolation methods are needed, and those due to Everett, involving central differences of various even orders, were adopted and analyzed.

For the planets and Sun the goal (1) has been achieved (except probably for Mercury, for short periods during the retrograde motion around inferion conjunction). In fact, going from the innermost to the outermost planets, successively wider intervals would be acceptable; but for convenience, only two interval were used: 5d where necessary (for Mercury an Venus), 10d otherwise.

For the Moon, the fastest-moving body, even 5day-intervals are not short enough to permit accurate inter- polation, and we must be satisfied with less. Fortunately the uncertainties due to interpolation (and also due to neglected perturbations) for the Moon are equivalent of only a few hours in time, so that only for the most critical cases, such as occultations, would recomputation of a lunar position be necessary.

If t is the desired instant, and to and t1 are the tabular instants which immediately precede and succeed it, then *x = (t - to) / (t1 - to)* is the *argument of interpolation*; it lies between 0 and 1; *t1 - to* is either 5d (for Moon, Mercury, Venus) or 10d (for Sun, Mars, Jupiter, Saturn). Table 1.4.1 gives values of *x* for various *(t - to)* in integral days and hours, for *t1 - to* = 5d and 10d.

Example. PVN's test case (27 Oct., -6y, 6 p.m., Memphis). We have seen that this instant is given by

t = *t* tab = 6Y, Oct. 21 + 5d 23h.91
= - 6L Oct. 26 + 23h.91

Then for five-day bodies,

x = (*t* - *to*) / (*t1* - *to*) = 23h.91 / 5 d = 0.19925 (between Oct. 26 and Oct. 31)

and for ten-day bodies

x = (*t* - *to*) / (*t1* - *to*) = 5 d 23h.91 / 10 d = 0.599625 (between Oct. 21 and Oct. 31)

For many purposes, linear interpolation (discussed in the next section) will be sufficiently accurate.

Greater accuracy (to nearly the full precision of the tables, except in the case of the Moon), can be achieved by higher-order methods.

TABLE 1.4.1.

Arguments of interpolation

These tables give *x = (t - to) / (t1 - to), for (t - to)* in integral days or hours, and (*t - to*) either five days or ten days, to sufficient precision to preserve one guard figure in the results ($0^o.00_1$ for the Sun and planets, $0^o.0_1$ for the Moon).

For greater precision, 1h / 5d = 1/120=0.0083333 .... 1h / 10d = 1/240 = 0.0041666....

	t1 - to	t1 - to		t1 - to	t1 - to
t - to	5d	10d	t - to	5d	10d
0d	.0000	.0000	0h	.0000	.0000
1	.2000	.1000	1	.0083	.0042
2	.4000	.2000	2	.0167	.0083
3	.6000	.3000	3	.0250	.0125
4	.8000	.4000	4	.0333	.0167
5	1.0000	.5000	5	.0417	.0208
6	---	.6000	6	.0500	.0250
7	---	.7000	7	.0833	.0292
8	---	.8000	8	.0667	.0333
9	---	.9000	9	.0750	.0375
10	---	1.000	10	.0833	.0417
			11	.0917	.0458
			12	.1000	.0500
			13	.1083	.0542
			14	.1167	.0583
			15	.1250	.0625
			16	.1333	.0667
			17	.1417	.0708
			18	.1500	.0750
			19	.1583	.0792
			20	.1667	.0833
			21	.1750	.0875
			22	.1833	.0917
			23	.1917	.0958
			24	.2000	.1000

1.4a. LINEAR INTERPOLATION

If *yo* and *yx* are the values of a function (e.g., longitude or latitude) at *to* and *T1*, and if *x* is the argument of interpolation, then the linear interpolate is

$$yLin = yO + X(y1 - yo)\ (+ - e/2)$$

or

$$yLin = yO + X\, dy\ \ (+ - e/2)$$

where

$$dy = y1 - y0$$

Since *yo, yx* are each subject to rounding errors of up to *e/*2 (where *e* is the tabular unit: $0^o.01$, except $0^o.1$ for the Moon), the same error of *e*/2 is inherent in yLin. (If higher-order interpolation is to be done, the linear interpolate will be a starting point; and to avoid unnecessary loss of accuracy it is well to keep one or two guard figures - i.e., more figures than the tabular precision - until the final rounding to tabular precision.)

Examples. For his test date, PVN computed positions for Sun, Mars, Jupiter, Saturn, and Moon. Here we compute the corresponding linear interpolates from the present tables, for the longitudes of those bodies. *Sun*. Here *yo*=$206^o.22$ (for 21 Oct., -6), *y*1 = $216^o.36$ (for 31 Oct.,-6), hence *dy* = $+10^o.14$; *x* = . 599625; hence

$$YLin = Yo + x.\, dy = 206^o.22 + (.599625)(+ 10^o.14)$$
$$= 206^o.22 + 6^o.080_{02}$$
$$= 212^o.30_{02}\ \ (+ - 0^o.00_{50})$$
$$= 212^o.30 \text{ (rounded).}$$

Mars, Jupiter, Saturn. Similarly we find,

for Mars YLin = 263º-04$_{11}$ (+ - 0º.00$_{50}$)
for Jupiter YLin = 345º.18$_{42}$ (+ - 0º.00$_{50}$)
for Saturn YLin = 345º.79$_{22}$ (+ - 0º.00$_{50}$)

Moon. Here the tabular interval is 5d; we have *Yo* = 258º.3 (26 Oct., -6); *y1* =325º.1 (31 Oct., -6); *Yx* - *Yo* = + 66º.8; *x* = .19925;

YLin = 258º.3 + (.19925) (+ 66º.8)
= 258º.3 + 13º.3$_{10}$
= 271º.6$_{10}$ (+ - 0º.0$_{50}$)
= 271º.6 (rounded)

The maximum errors (exclusive of rounding) committed by relying on linear interpolation (over five- or ten-day intervals as the case may be) are estimated in the upper part of table 1.4b.1.

1.4b. HIGHER-ORDER INTERPOLATION

If intertabular values are desired to greater accuracy than obtainable by linear interpolation, higher-order methods must be used. The Everett methods, which use central even differences [5] to supply a correction to the linear interpolate, are recommended for convenience.

Everett's interpolation formula may be written

$$Y(x) = \text{YLin} + E_o^{(2)}(x) . D_o^{(2)}(x) + E_1^{(2)}(x) . D_1^{(2)}$$
$$+ E_o^{(4)}(x) . D_o^{(4)}(x) + E_1^{(4)}(x) . D_1^{(4)}$$
$$+ E_o^{(6)}(x) . D_o^{(6)}(x) + E_1^{(6)}(x) . D_1^{(6)}$$
$$+ \ldots$$

where *x*, and YLin = *Yo* + *x*.dy, are as before; $D_o^{(2k)}$ and $D_1^{(2k)}$, often also written $dy^{(2k)}$yo, $dy^{(2k)}$y1, are the even central differences, as illustrated in the later example, and the Everett coefficients $E_o^{(2k)}(x)$ and $E_1^{(2k)}(x)$ are certain polynomial functions of their argument *x*.

The Everett methods are useful if the series converges, and especially if the convergence is so fast that only a few terms of the series provide a sufficiently good approximation. If the above series is truncated so that only terms through order 2*m* are included, the approximate formula,

$$y(x) \sim \text{YLin} + \ldots . E_o^{(2m)}(x) . D_o^{(2m)} \text{ and } E_1^{(2m)}$$

will be called Everett's formula of order 2*m* (it is an interpolation formula of order 2*m* + 1, since two differences of order 2*m* are used). An Everett's formula of a particular order may be useful, and bounds for its error can be estimated, even if the infinite series does not converge; but when there is non-convergence, one cannot obtain arbitrarily high accuracy of interpolation by sufficiently increasing the order of interpolation.

For the Sun and planets, and the chosen tabular intervals, the Everett series suffices (with the possible exception of Mercury in certain portions of its synodic period). In table 1.4*b*.1 are shown approximate bounds for the interpolation errors remaining after interpolation of the indicated order has been performed in the ephemeris. These estimates are based upon the usual error formulas, using the observed range of differences over reasonable periods of time.

TABLE 1.4b.1
Approximate bounds for interpolation errors remaining after interpolation of the indicated order in the ephemeris [Table for Five-day bodies; Ten-day bodies for the Moon, Mercury, Venus, Mars, Jupiter, Saturn, Sun]
[TABLE OMITTED]

For the Moon (and possibly at times for Mercury), owing to the shortness of the period compared to the tabular interval, the Everett's series does not suffice, and full tabular accuracy cannot be obtained by these methods; with higher-order methods, the rrros will eventually decrease.

Test calculations indicate possible interpolation errors of at least $0^o.5$ after 2nd order, $0^o.2_5$ after 4th order, and $0^o.1$ after any order of Everett interpolation. While interpolation of suitable order could probably give accuracy to a few $0^o.1$, an exact estimation of the necessary order, and of the maximum errors for various orders, did not seem warranted. The respective entries in table 1.4b.1 are therefore shown as *. For a discussion of additional accuracy for the Moon, see sections 1.5 and 1.6. To perform an Everett interpolation of order 2*m*, there are needed 4*m* + 2 equally spaced tabular values, namely the *m* + 1 values just on each side of the desired instant.

Example. *The PVN test date for Mars*. It was shown earlier that the PVN test date lies a fraction *x* = 0.599625 between the tabular dates of 21 Oct. and 31 Oct., -6. Since Everett interpolation of order 4 (= 2*m*) is occasionally needed for Mars longitudes, we would form a table of 4 + 2 = 6 (= 2*m* + 2) longitudes, 6/2 = 3 (= *m* + 1) on each side of the desired instant, and form the 4th differences. To show the general behavior of the differences, however, in this example we use two additional values on each side. In table 1.4*b*.2, the values lying within the stepped lines are th ones actually needed; those lying outside are the addi tional ones (and their differences) not needed. Differences higher than the 4th could be computed, bu they are so small as not to contribute to the interpolate, value, and so irregular, owing to cumulated roundind errors, as to be meaningless.

TABLE 1.4b.2

Mars longitudes, near PVN test date, 0th to 4th difference

[TABLE OMITTED]

The entries t be used in the interpolation formula are enclosed in boxes, (Actually, if l~y =+$7^o.69$ is used, then Yx $266^o.12$ is not; but it is included for symmetry.)

The values of interest are *Y*o = $258^o.43$, ay = + $7^o.69$,

$D_0^{(2)} = +0^o.07,\ D_0^{(2)} = +0^o.05,\ D_0^{(4)} = +0^o.01,\ D_1^{(4)} = +0^o.01.$

The linear interpolate is

$$Y\text{Lin} = yo + x \cdot 8y\text{: } 258^o.43 + (0.599625)\cdot(7^o.69)$$
$$= 258^o.43 + 4^o.61_{11}$$
$$= 263^o.04_{11}$$

as found earlier.

Since the 4th differences are too small to be significant (their products with the Everett coefficients would be negligible), we ignore them, and do a second-order Everett interpolation. For x = 0.599625, the second-order Everett coefficients are found from standard tables to be

$E_o^{(2)}(x) = -.0560,$
$E_1^{(2)}(x) = -.0640.$

Then the correction is

$$E_o^{(2)}\cdot D_o^{(2)} + E_1^{(2)}\cdot D_1^{(2)} = (-.0560)\cdot(+0^o.07) + (-.0640) - (+0^o.05)$$
$$= -00^o.00_{39} - 0^o.00_{32}$$
$$= -0^o.00_{71}.$$

Added to the linear interpolate this gives

$$Y \text{ 2nd Ev.} = 263^o.04_{11} - 0^o.00_{71}$$
$$= 263^o.03_{40}.$$

This has an uncertainty of up to about $0^o.00_{50}$ from the similar uncertainty, owing to rounding, in *yo* and *yx.*

A separate 704-test calculation, to extra precision, for the exact instant in question, yielded $263^o.03_{45}$, agreeing with the Everett-interpolated value within 0.00_{45}. This corroborates the effectiveness of the interpolation method (although the very close agreement reached here is better than would be normally expected in view of the unavoidable rounding error of $0^o.00_{50}$).

(The above more precise value of 263º.03$_{45}$ exceeds the PVN value of 262º.98 by 0º.05$_{45}$. This difference has been adequately resolved into contributions due to (a) the use in this ephemeris of the "new" elements (see section 2.5), also used in the later PVN but not in his test case; (b) the inclusion in the ephemeris of perturbations, all omitted by PVN; and (c) a residue which is within the maximum possible effect in PVN's results, in this configuration, of the various PVN rounding errors.)

The contributions of the second-order terms, and of higher-order terms, may in other cases, principally for Mercury, be appreciably larger than in this example, as indicated by the error estimates given earlier. In case of doubt, if the maximum precision obtainable from the ephemeris is desired, it is well to interpolate to the full order indicated by those estimates, at least until familiarity with the contributions of the various orders is obtained. '

Similarly applying second-order Everett corrections to all the linear interpolates (except. the Moon) of the preceding section, we have in all, for the ephemeris values of PVN's test cases: *Sun,* 212º.29$_{61}$; *Mars*, 263º.03$_{40}$; *Jupiter*, 345º.14$_{56}$; *Saturn*, 345º.77$_{35}$. These are compared with PVN's values in section 3.11.

1.5. PARALLAX

To an observer on the surface of the Earth, the position of a body at a finite distance will generally differ from its geocentric position, owing to the observer's location off the line 'from the center of the Earth to the body. This effect is called parallax. Since it depends not only on the position and distance of the body, but also on the observer's geographic latitude and longitude, and on the time of day, it has not been included in this ephemeris. Table 1.5.1, giving bounds for the parallax of the various bodies, shows that it can be of possible interest for ancient observations only in the case of the nearest body, the Moon, for which it can slightly exceed 1ø, the Moon's motion in about two hours. In cases where this quantity is of interest, the parallax may be computed by standard methods, to be found, for example, in Smart.

TABLE 1.5.1
Bounds for horizontal parallax

	o
Moon	1.0$_3$
Sun	0.00$_3$
Mercury	0.00$_5$
Venus	0.01$_0$
Mars	0.00$_7$
Jupiter	0.00$_1$
Saturn	0.00$_1$

1.6 THE MOON

For the Moon, the dominant limitation on accuracy is that practical interpolation methods are here limited in accuracy to one or a few 0º.1. It would, therefore, have been pointless to tabulate closer than 0º.1, or to include the smaller perturbations.

Linear interpolation in the Moon's ephemeris is good to about 1º.6, or 3 + hours, in longitude. The omitted parallax can amount to about 1º.0, or two hours. The total uncertainty from these is thus less than six hours, a fraction of a day.

If greater precision is desired, the parallax must certainly be computed, and this is presumed in what follows.

Higher-order interpolation in the ephemeris can probably reduce the interpolation error to one or two 0º.1, but the necessary order, and precise bounds, have not been clearly established. Alternatively, one could compute using PVN (A) (new coefficients), with a rounding error under 0º.1; this is especially easy, since the most laborious computation, the reduction from heliocentric to geocentric coordinates, is not necessary for the Moon. Either of these two methods still has an uncertainty of about 0º.2, or 2/5 hour, due to neglected perturbations.

For an additional decimal, appropriate to occultations, it will be necessary to refer to the original sources, and to include sufficient small perturbations.

[1] I.e.,with respect to the mean equinox of date.

[2] They are, specifically, the dates for which the Julian day numbers of the preceding Greenwich noons are multiples of 10: that of the first entry, 3 Jan. -600, is 1,501,910.

[3] This could be interpreted as the L.C.T. at longitude 240? West (ignoring the date line); but this point may be disregarded.

[4] This is a time for which PVN worked out the positions of several bodies as examples of his methods.

[5] For further details see a text on numerical analysis, e.g., Hildebrand, 103 ft.

[End of Part 1: Use of this Ephemeris (Tuckerman 1962:3-7)]

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